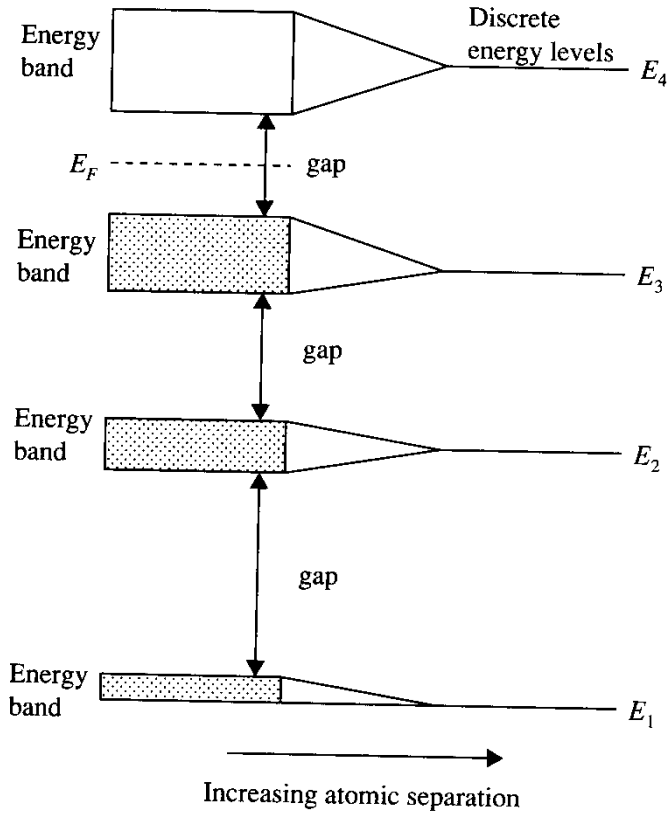
Lecture 2

Origin of SC Bandgap

- The discrete energy states of electrons in an atom are broadened from the individual atom states into energy bands as the atoms are brought closer together to form a crystal.
- When the bandgap energy is on the order of 2eV, room temperature thermal energy can break covalent bonds and accelerate ionized electrons into empty states in the crystal where they can conduct electrically -> Semiconductors



Energy Band Structures

- The wave function of an electron in any band is characterized by its wave function (ψ) and wave vector (k)

$$\Psi(\bar{r}) = u_k(\bar{r})e^{i\bar{k}\cdot\bar{r}}$$

$$\lambda_e = \frac{2\pi}{\left|\bar{k}\right|} = \frac{2\pi}{k}$$

- k can only take on a quantized set of values set by boundary conditions in the crystal, where the total phase shift of the electron wave vector across a crystal with dimension L_x, L_y, L_z is 2π and the volume of a unit cell in k-space is ΔV_k

$$k_i = \frac{2\pi}{L_i}s,\ s = 1,2,3,\ \ldots$$

$$\Delta V_k = \Delta k_x \Delta k_y \Delta k_z = \frac{(2\pi)^3}{L_x L_y L_z} = \frac{(2\pi)^3}{V}$$

Energy Band Structures

- The number of states in a spherical shell (*k*-space) of thickness d*k* and radius *k* is

Number of states within dk = Number of states per unit volume of shell dk

$$=\rho(\mathrm{k})\mathrm{d}\kappa=2\frac{4\pi\mathrm{k}^2}{\Delta V_k}\mathrm{d}\kappa=2\frac{4\pi\mathrm{k}^2}{(2\pi)^3/V}\mathrm{d}\kappa=\frac{V\mathrm{k}^2}{\pi^2}\mathrm{d}\kappa$$

- The energy of an electron with effective mass m_c and wave vector *k* in the conduction band is (we will assume energy depends on magnitude k and not direction)

$$E_c(\bar{k})=\frac{\hbar^2k^2}{2m_c}$$

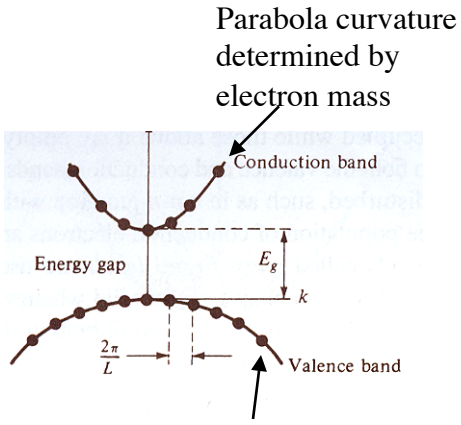
- The number of electron and holes per unit volume as a function of energy ($\rho_c(E)$, $\rho_h(E)$) is found from the number of available states per energy level per unit volume

$$\rho(E)dE=\frac{1}{V}\rho(k)dk$$

$$\rho_c(E)=\frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}E^{1/2}$$

$$\rho_c(\omega)=\hbar\rho_c(E)=\frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}\omega^{1/2}$$

$$\rho_h(\omega)=\hbar\rho_h(E)=\frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}\omega^{1/2}$$

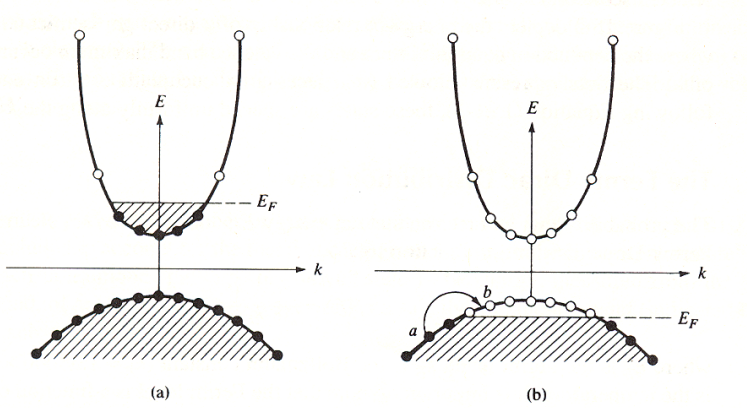


Fermi-Dirac Statistics

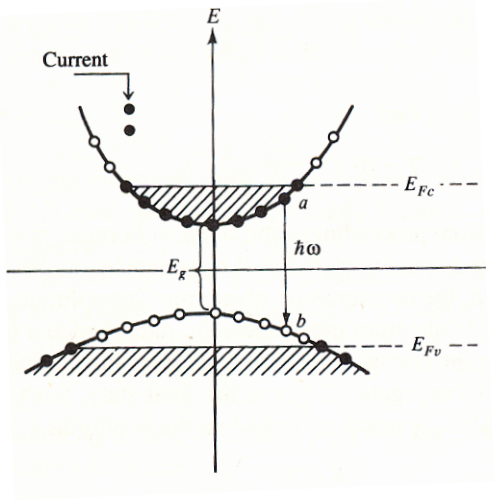
- The probability that a state at energy (E) is occupied by an electron is given by

$$f(E) = \frac{1}{e^{(E-E_F)/\kappa_B T} + 1}$$

- Doping the semiconductor can move the Fermi level into the conduction or valance bands depending on the doping concentration



- Disturbing the SC from TE, for example with current injection, will create Quasi-Fermi levels E_{Fc} and E_{Fv}

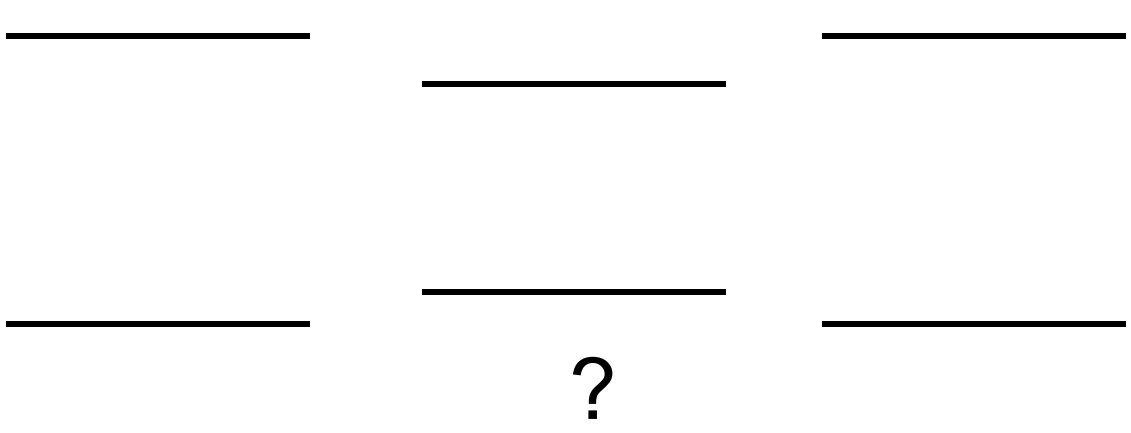


Guidelines

- Figure out bandgaps. (draw horizontal where there is no depletion).
- Figure out conduction band and valence band discontinuity.
- Figure out Fermi level for each material.
- Draw flat band first, then flatten out the Fermi level.
- Keep the band separation constant for any region where the bandgap is not changing.

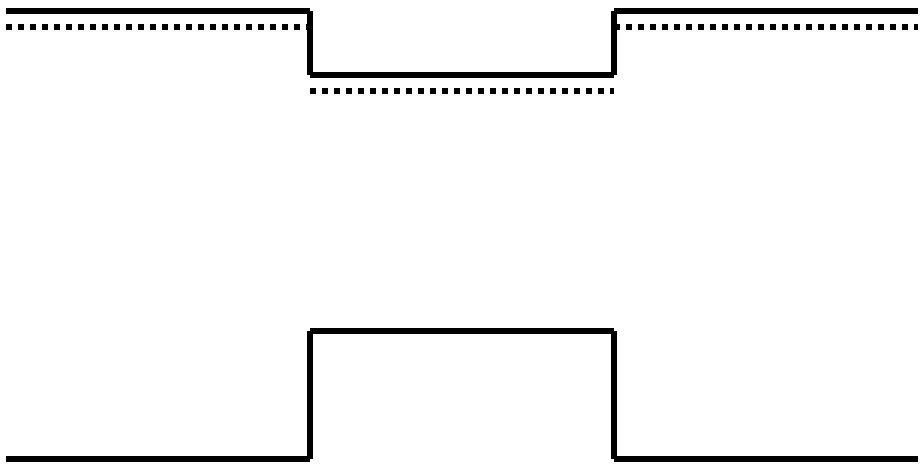
InP/InGaAs/InP Square Well

- N+ InGaAs: E_g=.76 eV
- N+ InP: E_g=1.35 eV



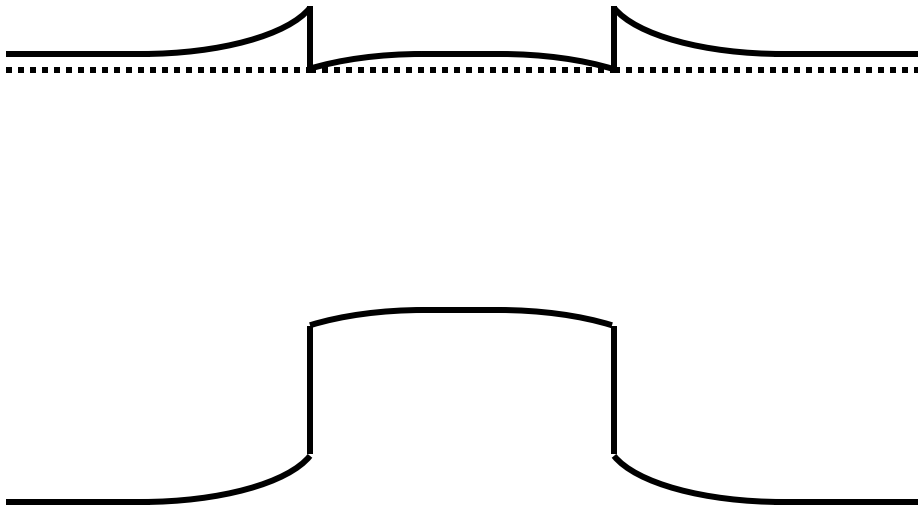
InP/InGaAs/InP Square Well

- N+ InGaAs: E_g=.76 eV
- N+ InP: E_g=1.35 eV
- ΔE_g=.59 eV
- ΔE_c=.25 eV
- ΔE_v=.34 eV
- Draw the flatband diagram first, ignoring doping.



InP/InGaAs/InP Square Well

- N+ InGaAs: E_g=.76 eV
- N+ InP: E_g=1.35 eV
- ΔE_g=.59 eV
- ΔE_c=.20 eV
- ΔE_v=.39 eV



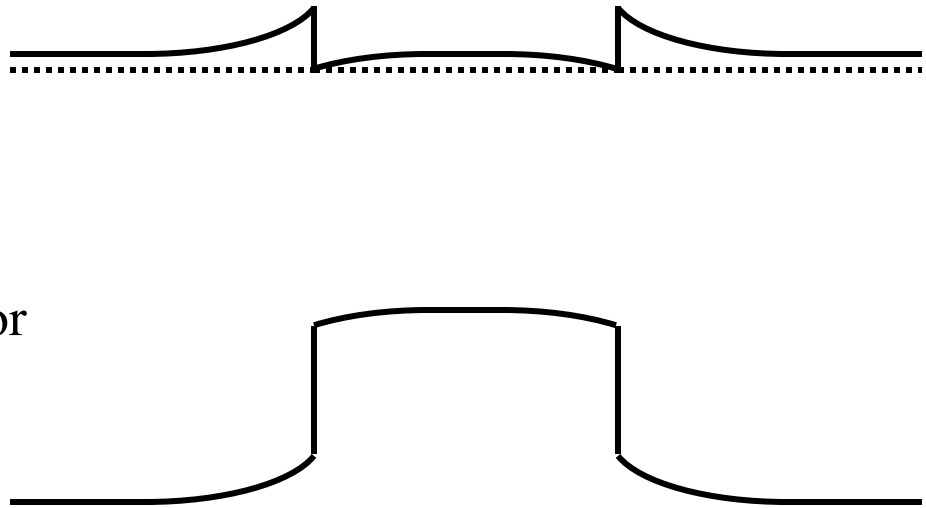
General comments

- If the composition is constant, the bandgap is constant. Hence, the separation of conduction band and valence band are constant.
- If there is no field, the bands are horizontal.
- Use the depletion edge approximation; either the material is depleted of free carriers (and the bands are bent) or there is no field and the bands are flat.
- Depleted doped material has a quadratic bend and linearly increasing field.
- Depleted undoped material has constant electric field and linear band bending.

$$\frac{d^2\Phi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{p - n + N_D - N_A}{\varepsilon}$$

InP/InGaAs/InP Square Well

- N+ InGaAs: E_g=.76 eV
- N+ InP: E_g=1.35 eV
- ΔE_g=.59 eV
- ΔE_c=.20 eV
- ΔE_v=.39 eV
- Adjust Fermi level to account for bias.
- Keep bandgaps constant.



Calculate the absorption bandedge assuming a 100 Angstrom quantum well and m_e*=0.1 m_e and m_h*=1 m_e
Calculate the absorption bandedge assuming a 100 Angstrom quantum dot and m_e*=0.1 m_e and m_h*=1 m_e
Check the validity of your assumptions.

1D case

For simplicity, assume an infinite square well, then see if that is valid.
Electron and hole levels:

$$E = \frac{\hbar^2}{2m^*}\left(\frac{n\pi}{a}\right)^2 + \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2)$$

$$1D\ electron:$$

$$E_1 = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{a}\right)^2 = 0.03eV$$

$$\Delta E_c = 0.2eV$$

$$1D\ hole:$$

$$E_1 = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{a}\right)^2 = 0.003eV$$

$$\Delta E = h\upsilon = \frac{hc}{\lambda} = .76 + .03 + .003 = .79eV$$

$$\lambda = \frac{1.24eV\mu m}{.79eV} = 1.57\mu m$$

3D case

For simplicity, assume an infinite square well, then see if that is valid.
Electron and hole levels:

$$E = \frac{\hbar^2}{2m^*}\left(\left(\frac{n_x\pi}{a}\right)^2 + \left(\frac{n_y\pi}{b}\right)^2 + \left(\frac{n_z\pi}{c}\right)^2\right)$$

$$3D\ electron:$$

$$E_1 = \frac{3\hbar^2}{2m^*}\left(\frac{\pi}{a}\right)^2 = 0.09eV$$

$$3D\ hole:$$

$$E_1 = \frac{3\hbar^2}{2m^*}\left(\frac{\pi}{a}\right)^2 = 0.009eV$$

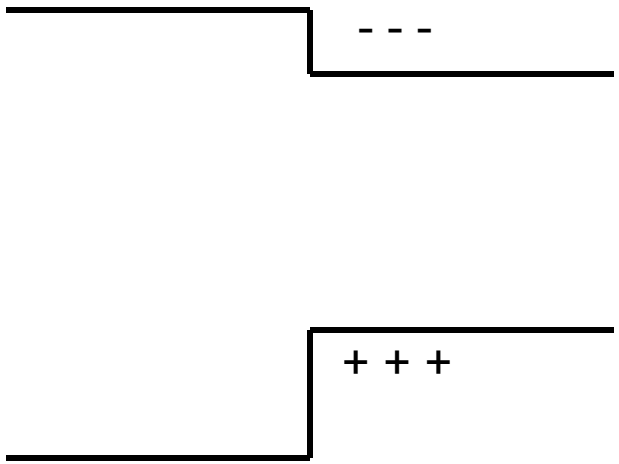
Infinite square well approximation poor, Even for the lowest electron energy level.

$$\Delta E = h\upsilon = \frac{hc}{\lambda} = .76 + .09 + .009 = .86eV$$

$$\lambda = \frac{1.24eV\mu m}{.86eV} = 1.44\mu m$$

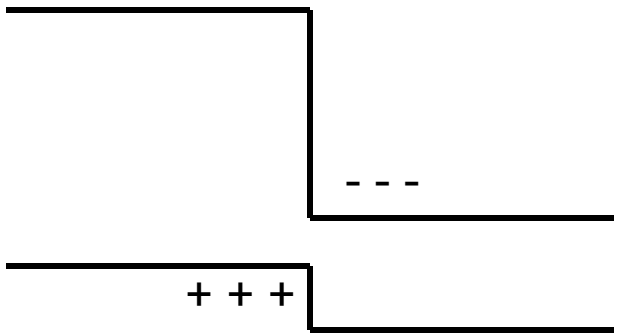
Three types of heterojunctions

- Type I (Straddling)
  - Free electrons and holes reside in the same region of space.
  - Examples: AlAs/GaAs, InP/GaInAs



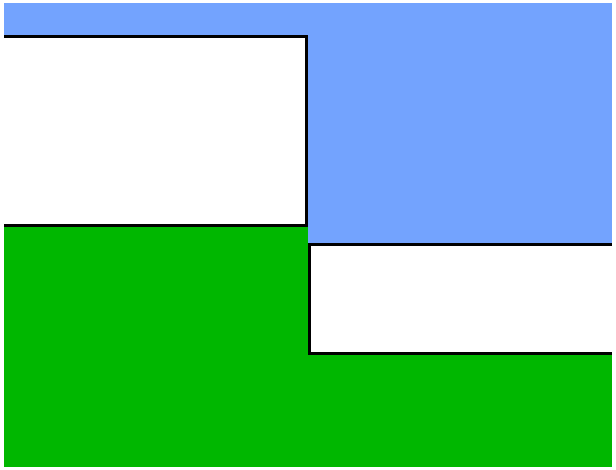
Three types of heterojunctions

- Type I (Straddling lineup)
- Type II (Staggered lineup)
  - ΔE_c and ΔE_c have the same sign.
  - Free electrons and holes reside in different regions of space.
  - Example: AlSb/InAs



Three types of heterojunctions

- Type I (Straddling lineup)
- Type II (Staggered lineup)
- Type III (Broken Gap Lineup)
  - ΔE_c and ΔE_c have the same sign.
  - Free electrons and holes reside in different regions of space.
  - Example: GaSb/InAs



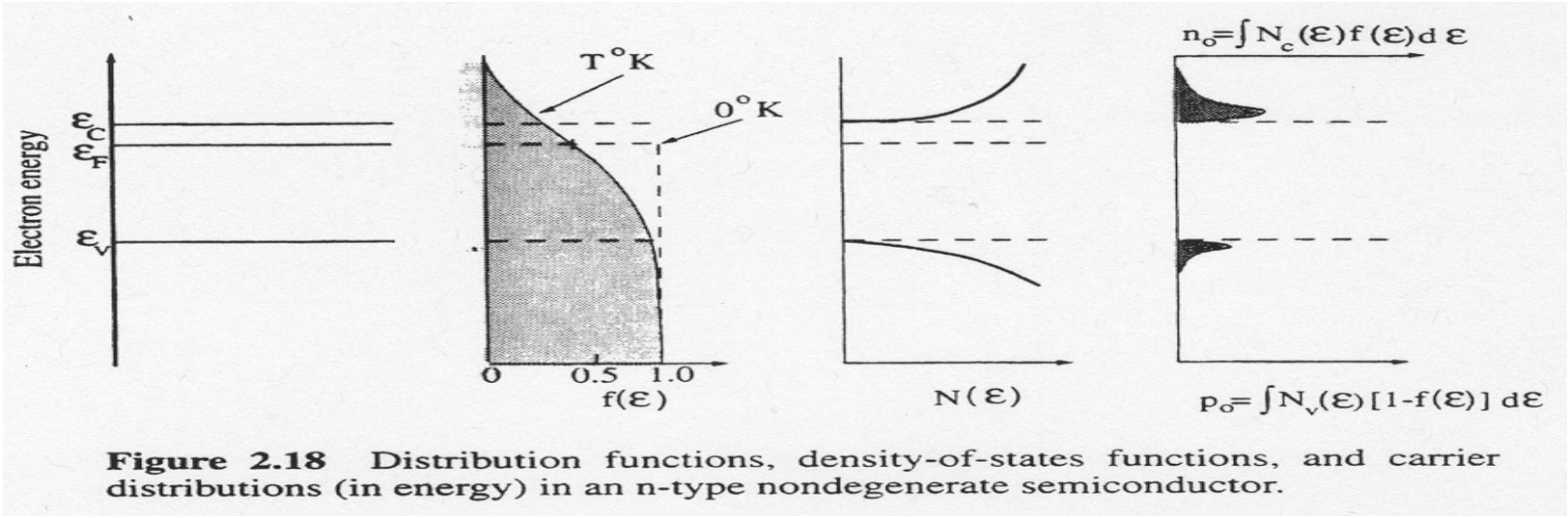


Figure 2.18 Distribution functions, density-of-states functions, and carrier distributions (in energy) in an n-type nondegenerate semiconductor.

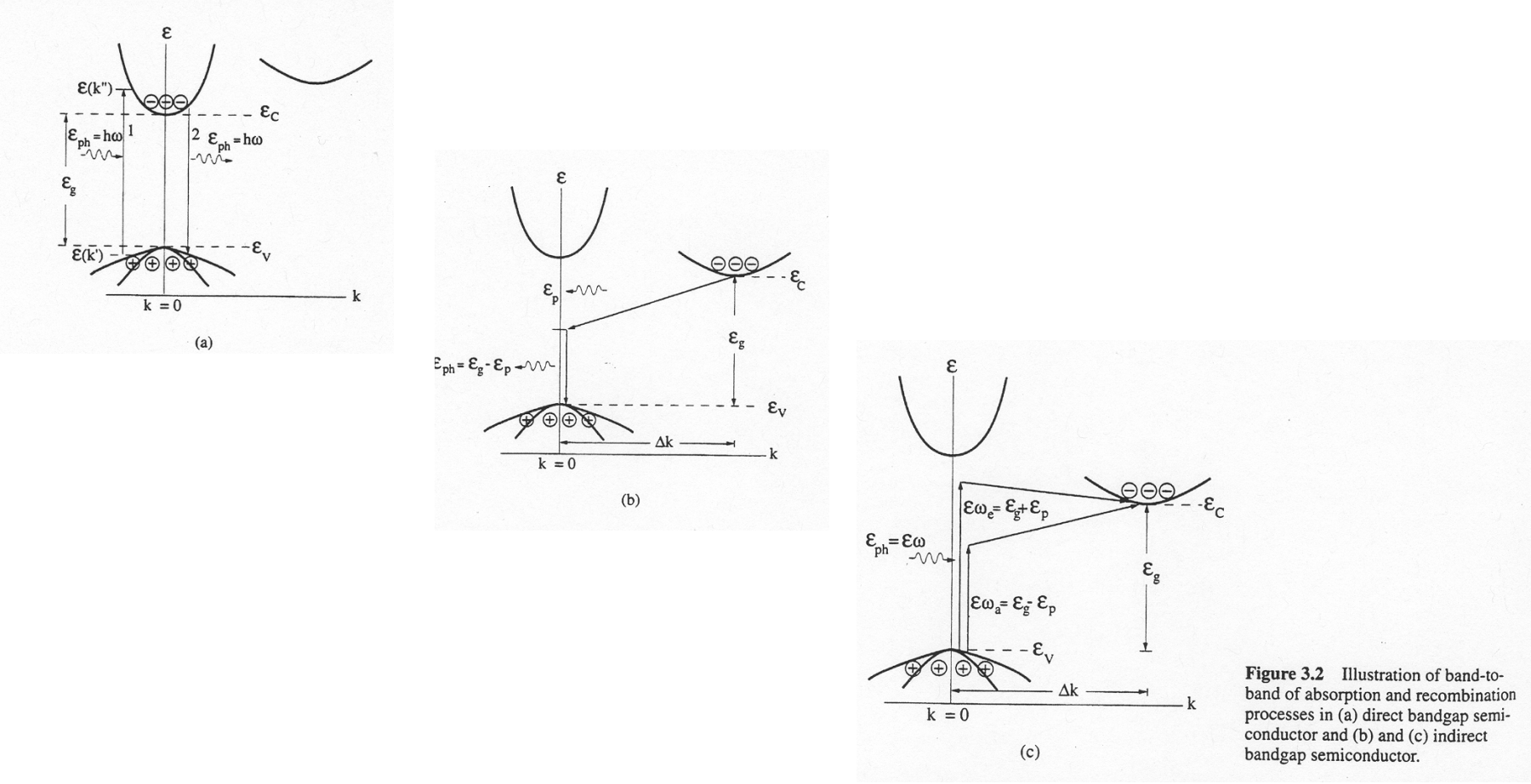
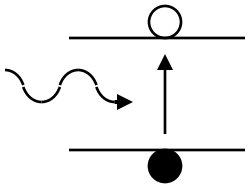


Figure 3.2 Illustration of band-to-band of absorption and recombination processes in (a) direct bandgap semiconductor and (b) and (c) indirect bandgap semiconductor.

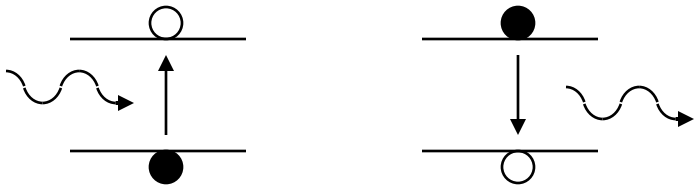
Electronic transitions



R_{12}

- R_{12}: absorption of a photon

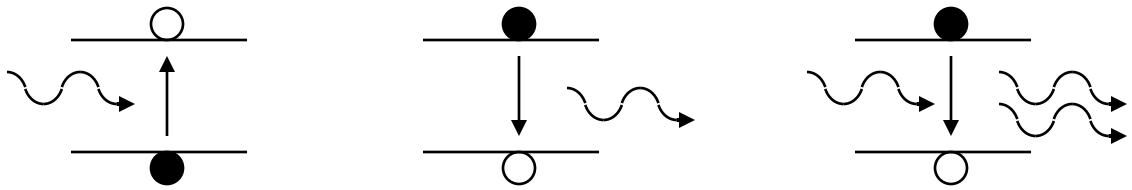
Electronic transitions



- R_{sp}

- R_{12}: absorption of a photon
- R_{sp}: spontaneous emission of a photon

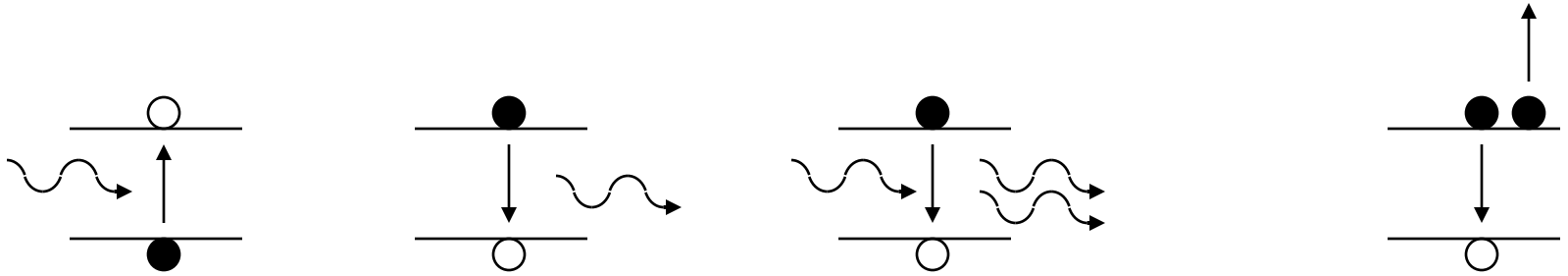
Electronic transitions



- R_{21}

- R_{12}: absorption of a photon
- R_{sp}: spontaneous emission of a photon
- R_{21}: stimulated emission of a photon

Electronic transitions



R_{nr}

- R_{12}: absorption of a photon
- R_{sp}: spontaneous emission of a photon
- R_{21}: stimulated emission of a photon
- R_{nr}: nonradiative recombination (Auger, trap, etc.)

Rate Equations

$$\frac{dN}{dt} = G - R$$

N is the electron density (assumed equal to hole density)

G is the generation rate of electrons

R is the total recombination rate

Rate Equations

$$\frac{dN}{dt} = G - R$$

$$G = \frac{\eta_i I}{qV}$$

$$R = R_{sp} + R_{nr} + R_i + R_{st}$$

$$R = BN^2 + AN + CN^3 + R_{st}$$

$$R \approx \frac{N}{\tau}$$

N is the electron density (assumed equal to hole density)

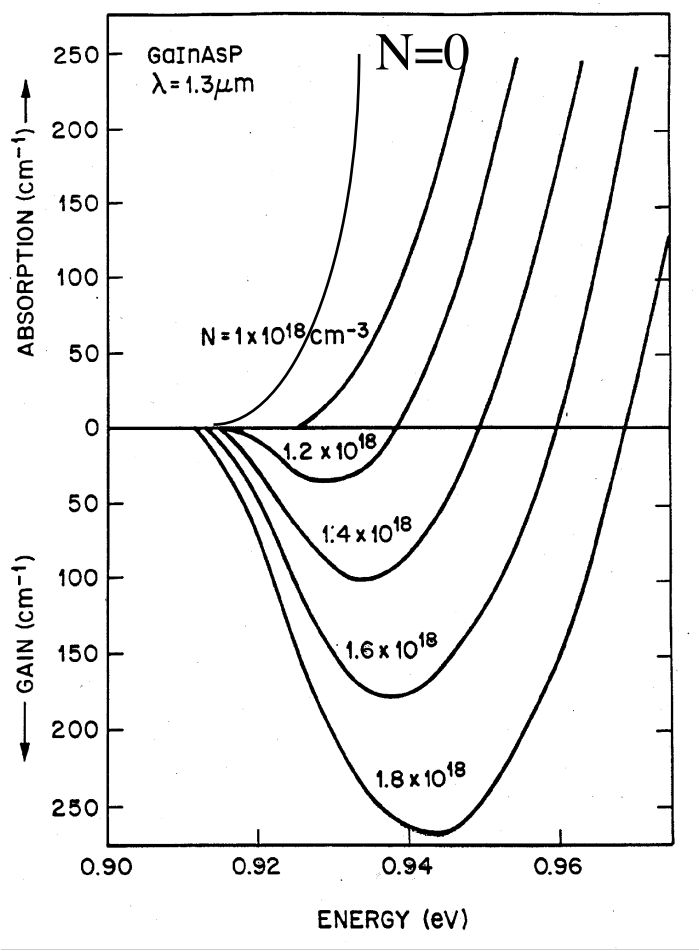
G is the generation rate of electrons

R is the total recombination rate

Material Gain

Calculated gain curves for InGaAsP/InP laser operating at 1.3μm

- Gain peak moves to shorter wavelengths with higher pumping
- Higher differential gain for wavelengths shorter than the gain peak



N.K.. Dutta, J. Appl. Phys., **51**, 6095 (1980)

ECE 162C: PROBLEM SET #1
DUE Tuesday, APRIL 18, 2006

Problem 1:

An electron is trapped in a one dimensional potential well 5 nm wide and 100 meV deep. How many bound states exist? What are the energy levels of the first three levels relative to the well bottom? Assume an electron mass equal to 0.1 the free electron mass.

Problem 2:

An electron is trapped in an InGaAs quantum well 5 nm wide. How many bound states exist? What are the energy levels of the first three levels relative to the well bottom? Repeat the calculations for a bound hole. What is the wavelength of the lowest energy emission?

Problem 3:

Plot the carrier density versus the quasi Fermi level for the conduction band in bulk InGaAsP at 300 K. Cover the carrier density range from 1017 to 1019 cm^{-3} and use a logarithmic scale for the carrier density axis.